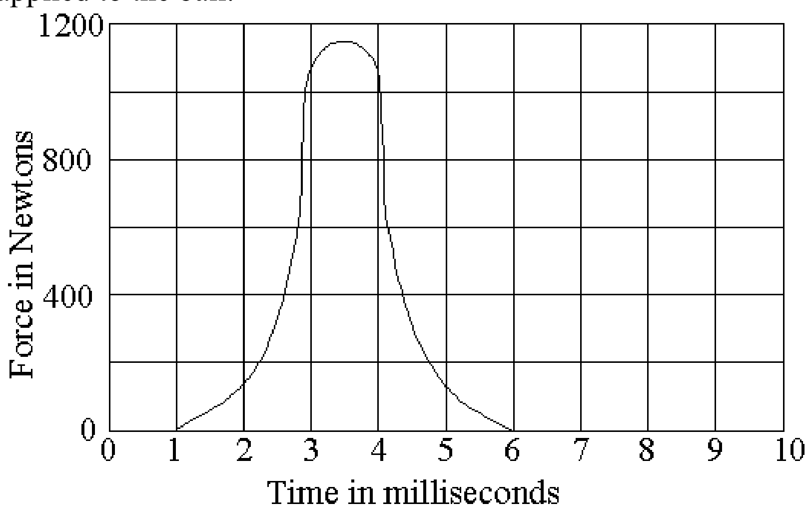
# Physics

Lesson Plan #8
Impulse and Momentum
David V. Fansler
Beddingfield High School

Impulse and Momentum

*Objectives: Compare the system before and after an event in momentum problems; Define the momentum of an object; Recognize that impulse equals the change in the momentum of an object.*

- Impulse and Momentum
  - What happens to a tennis ball when a tennis racket strikes it on a serve? The ball is lobed into the air with minimal horizontal velocity, the server swings the racket, hitting the ball and the ball leaves the racket with a large velocity. The racket on the other hand maintains a fairly constant speed during the whole swing.
    - So how are the velocities of the ball before and after the collision related to the forces on the ball?
      - By Newton's 1st law, we know that if there are no forces on the ball that it remains at the same velocity (whether that is 0 or some value above 0) – it is constant.
      - By Newton's 2nd law we know that the velocity of the ball is changed by a net force acting on it.
      - Since the only horizontal force on the ball was the racket, the force must have been from the racket.
      - A graph of the force on the ball vs. time would show that there was only a brief moment (a few thousandths of a second) of force applied to the ball.




Impulse - Figure 1

- The graph shows how force changes over time – interesting to note that the typical maximum force on the ball is over 1000 times the weight of the ball.

- Relating Impulse and Momentum
  - Newton's 2nd law will help explain how the net force acting on it changes the momentum of an object.
  - Remember that Newton's 2nd law states $F = ma$, in which $a$ can be re-written to our original definition where $a = \frac{\Delta v}{\Delta t}$ to $F = m\frac{\Delta v}{\Delta t}$ and moving $\Delta t$ we get $F\Delta t = m\Delta v$
    - $F\Delta t$ is the product of the average force and the interval of time over which it acts. This product is called the **impulse**. It has units of N·s. The magnitude of impulse is equal to the area under a curve such as in figure 1.

- On the right side of the equation, we could expand it where $\Delta v = v_2 - v_1$ to be $mv_2 - mv_1$. The product of mass times velocity is termed **linear momentum,** (plural is momenta) with a symbol of **p**. So p=mv, and re-writing the right side of the equation we get $F\Delta t = p_2 - p_1$. This is called the **impulse-momentum theorem**.
  - The impulse on an object is equal to the change in momentum that it causes.
  - If the force were constant, it would simply be the product of the force times the time interval of the force. Most times the force is not constant and the impulse is found by using the average force times the time interval, or by finding the area under the curve of a force-time graph.
- So what was the change in momentum of the tennis ball?
  - We just learned that the impulse-momentum theorem that the change in momentum ($p_2$-$p_1$) is equal to the impulse ($F\Delta t$). The impulse can be calculated from the force-time graph. The area in fig 1 is roughly 1.4 N·s. If we expand the N units, the N·s = kg·m/s, so the momentum gained is 1.4 kg·m/s.
- How much momentum did the ball have after being struck?
  - Using $F\Delta t = p_2 - p_1$, we can rearrange this to be $p_2 = F\Delta t + p_1$ , so we can see that the final momentum would be the initial momentum plus the impulse. If the ball was at rest prior to being struck, then its final impulse would simply be equal to the impulse ($p_2 = F\Delta t + 0 \rightarrow p_2 = F\Delta t$) or $p_2 = mv = 1.4\ kg \bullet m/s$
- If we know all this, what was the ball's final velocity?

- If the ball had a mass of 0.060kg, then we can solve for it using

  $p_2 = mv$

  *rearranged to*

  $$v = \frac{p_2}{m} = \frac{1.4kg \bullet m/s}{0.060kg} = 23m/s$$

- You should note that since velocity is a vector, so is momentum, and likewise, since Force is a vector, so is impulse – so watch your signs!

- How impulse-momentum saves lives
  - A large change of impulse could be a small force over a long time, or it could be a large force over a short time. The latter is the case for a head on car collision
  - What happens when a crash suddenly stops a car? The occupants of the car experience an impulse that brings their momentum to 0. If the driver is unrestrained, then it is the steering wheel that exerts a large force against the driver for a short interval of time. An airbag increases the interval of time, thereby reducing the size of the force.
  - Look again at $F = m\frac{\Delta v}{\Delta t}$ - we know that for a given situation, that the mass is constant, that the change in velocity is the same, so the only thing we can do to reduce the force is to increase the time.
  - The purpose of an airbag is to increase the time it takes for a person to come to rest, therefore decreasing the force at any instant of time.

Sample Problem

A 2200 kg SUV traveling at 94 km/h (26m/s) can be stopped in 21 sec by gently applying the brakes, in 5.5 s in a panic stop, or in 0.22s if it hits a concrete wall. What is the average force exerted on the SUV in each of these stops?

If we were to calculate the momentum before and after the stops

$p_1 = mv_1 = 2200kg \bullet 26m/s = 5.7\ x\ 10^4\ kg \bullet m/s$

$p_2 = mv_2 = 2200kg \bullet 0m/s = 0kg \bullet m/s$

Applying the impulse-momentum theorem to obtain the force needed to stop the SUV

$F\Delta t = p_2 - p_1$

$F\Delta t = 0m/s - 5.7\ x\ 10^4\ kg \bullet m/s$

$F\Delta t = -5.7\ x\ 10^4\ kg \bullet m/s\ or$

$$F = \frac{-5.7\ x\ 10^4\ kg \bullet m/s}{\Delta t}$$

Now we just have to substitute the times to solve for the force:

$$F = \frac{-5.7 \, x \, 10^4 kg \bullet m/s}{21s} = -2700N$$
$$F = \frac{-5.7 \, x \, 10^4 kg \bullet m/s}{5.5s} = -10000N$$
$$F = \frac{-5.7 \, x \, 10^4 kg \bullet m/s}{.22s} = -260000N$$

Another way would be using $F = m\frac{\Delta v}{\Delta t}$

For 21s we would have $F = 2200kg\frac{0m/s - 26m/s}{21s} = -2700N$

For 5.5s we would have $F = 2200kg\frac{0m/s - 26m/s}{5.5s} = -10000N$

For .22s we would have $F = 2200kg\frac{0m/s - 26m/s}{.22s} = -260,000N$

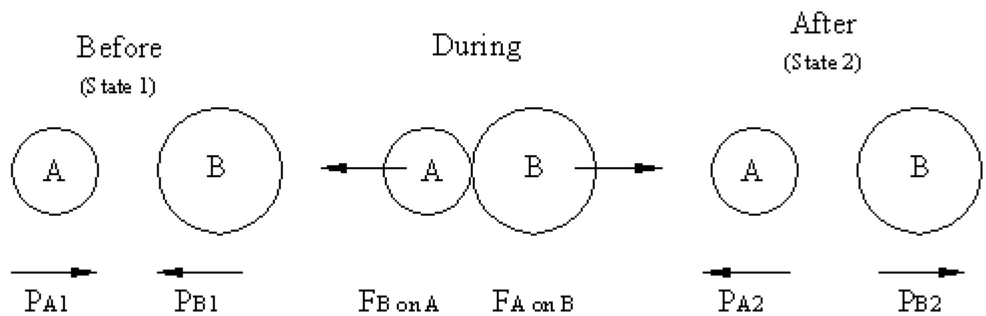
- Angular Momentum
  - As we saw when dealing with circular motion, the speed of an object changes only if torque is applied to the object. The quantity of motion used with rotating objects is called **angular momentum**.
  - Angular momentum of an object changes when the torque acts on the object.
  - For linear momentum, we saw the formula was *p=mv*. Angular momentum has a little longer definition – Angular momentum is a product of the object's mass, how far it is from the center of rotation( displacement), and the object's component of velocity perpendicular to the displacement.

The Conservation of Momentum

*Objectives: Relate Newton's 3rd law of motion to conservation of momentum in collisions and explosions; Recognize the conditions under which the momentum of a system is conserved; Apply conservation of momentum to explain the propulsion of rockets; Solve conservation of momentum problems in two dimensions by using vector analysis.*

- Two-Particle Collisions
  - In Newton's 3rd law, we learned that forces always come in pairs. When the tennis racket hit the tennis ball and imparted a large momentum on the tennis ball, was the momentum of the tennis racket also changed?
  - In reality, there is more than just the racket to consider – after all the racket was held in the hand of the player, which was attached to his arm, which is part of his body that is standing on the ground – but for simplicity we ignore all these other items and consider just the ball and the racket
  - When two balls collide, regardless of their size or masses, they exert and equal and opposite force on each other.
    - $F_B$ on $_A$ = -$F_A$ on $_B$

- Because they are in contact for the same amount of time, how do the impulses received by each ball compare? They must be equal in magnitude and opposite in direction.
- So how would the momenta of the balls compare after the collision? According to the impulse-momentum theorem, the final momentum, for each ball, is equal to the initial momentum plus the impulse.
  - For ball A: $p_{A2} = F_B \text{ on }_A \Delta t + p_{A1}$
  - For ball B: $p_{B2} = F_A \text{ on }_B \Delta t + p_{B1}$




Impulse - Figure 2

  - If we go back to $F_B \text{ on }_A = -F_A \text{ on }_B$ and substitute we get $p_{A2} = -F_A \text{ on }_B \Delta t + p_{A1}$
  - So now if we the momenta of the two balls, we get

$$\begin{array}{rl} & p_{A2} = -F_A \text{ on }_B \Delta t + p_{A1} \\ + & \underline{p_{B2} = F_A \text{ on }_B \Delta t + p_{B1}} \\ & p_{A2} + p_{B2} = p_{A1} + p_{B1} \end{array}$$

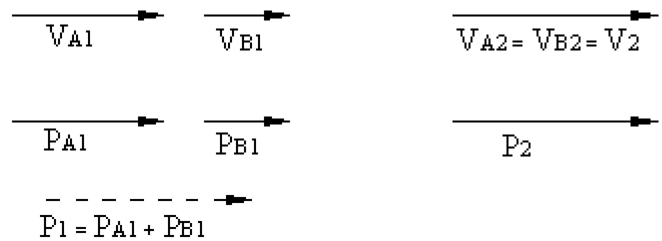
  - which simply shows that the sum of the momenta of the balls before the collision is the same as the sum of the momenta of the balls after the collision.
  - Another way of looking at it is the momentum lost by ball 1 is gained by ball 2 – momentum is conserved

- Momentum in a Closed System
  - A system is said to be **closed system** if no mass is gained or lost.
  - All forces are in a closed system are **internal forces**, and these are the only forces on the system
  - All forces outside the system are declared to be **external forces**
  - When the net external force is 0, a system is described as an **isolated system**
  - Under these conditions, the **law of conservation of momentum** is honored, where it states that the momentum of any closed system does not change as long as there are no net external forces

Sample Problem

A 2275kg car is going 28m/s and rear-ends an 875kg compact car going 16m/s on ice in the same direction. The two cars lock bumpers. How fast does the wreckage move immediately after the collision?

| *Known* | *Unknown* |
|---|---|
| $m_A = 2275kg$ | $v_2$ |
| $v_{A1} = 28m/s$ | |
| $m_B = 875kg$ | |
| $v_{B1} = 16m/s$ | |




Impulse - Figure 3

The law of conservation of mass applies, since the ice tends to make the external forces (friction) nearly 0, so

$P1 = P2$

$P_{A1} + P_{B1} = P_{A2} + P_{B2}$

$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$

Since the two cars stick together after the collision, then we can use this to state

$v_{A2} = v_{B2} = v_2$

$m_A v_{A1} + m_B v_{B1} = (m_A + m_B)v_2$

and solving for $v_2$ we get

$$v2 = \frac{m_A v_{A1} + m_B v_{B1}}{m_A + m_B} = \frac{(2275kg)(28m/s) + 875kg(16m/s)}{2275kg + 875kg} = 25m/s$$

- Rocket Engines in Space
  - How does a rocket change its velocity in space? The simple answer is that the rocket engine fires and makes the rocket accelerate. But let's look at it from the concepts we have learned about momentum.
    - A rocket engine carries fuel and an oxidizer (oxygen for burning the fuel). When these mix and burn, they resulting hot gases are expelled from the exhaust nozzle at high speed.
    - Since the rocket and chemicals are the system, then the system is closed.
    - The forces that expel the gases are internal forces, so the system is also isolated
    - Therefore the law of conservation of mass can be applied to this situation