Physics

Lesson Plan #7
Forces & Motion in Two Dimensions
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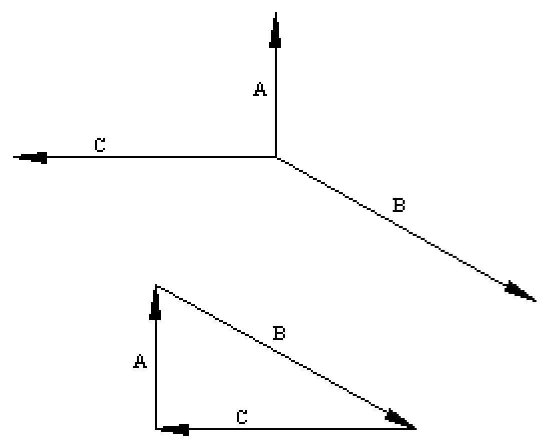
Forces in Two Dimensions

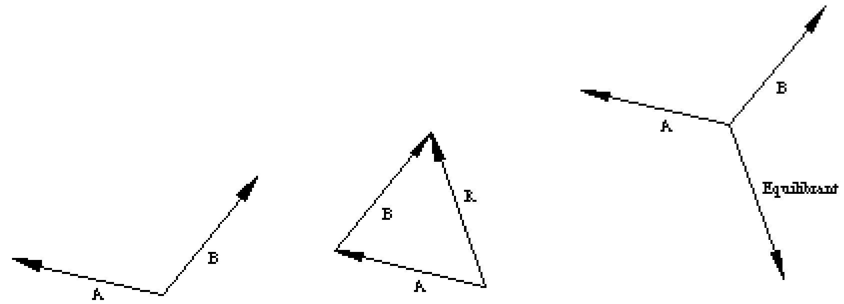
Objective – Determine the force that produces equilibrium when three forces act on an object; Analyze the motion of an object on an inclined plane with and without friction

- Equilibrium and the Equilibrant
    - In the last chapter we dealt with multiple forces working an object, but always at right angles to each other. Life is not so simple. What if we had a point with three forces acting on it?



Forces in 2D - Fig. 1

- Using the familiar vector diagram we have forces A, B and C working on a point. Is the point at rest? Remember that Newton's 2nd law is $a = \frac{F_{net}}{m}$, so if F_{net} is 0 then the forces are balanced and the object is at rest, or moving with constant velocity.
- Using vector addition, we can rearrange the vectors, putting the head of one to the tail of the next – in the case above this gives us a closed triangle indicating that the forces are balanced. The object is indeed at rest or in constant motion.
- What if you had two forces acting on an object. Could you add a third force to bring the object to equilibrium? The answer is yes, and this third force that brings the system to equilibrium is called an **equilibrant**
- From a vector diagram with two forces, we move the vectors around to solve for the resultant
- The equilibrant is equal in magnitude to the resultant, but opposite in direction of the resultant.
- This process would work for as many vectors as you have.



2 forces, unbalanced 2 forces with resultant 2 forces, balanced by equilibrant

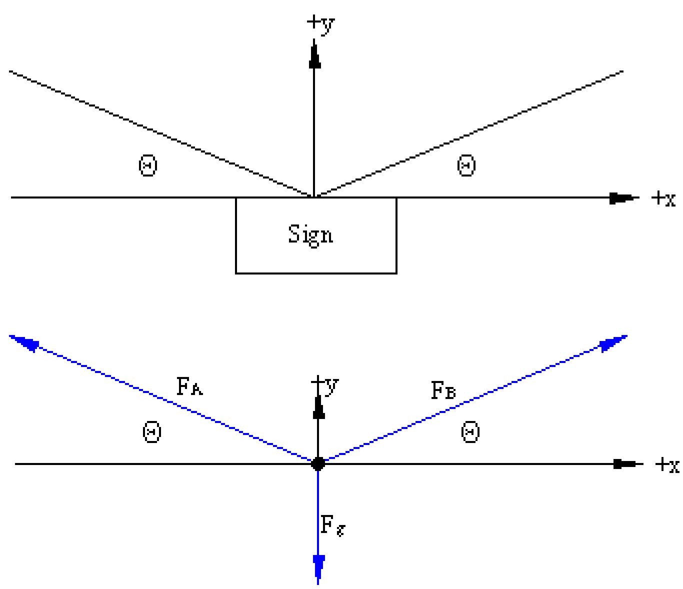
Forces in 2D - Fig. 2

Example Problem:

A 168 N sign is supported in a motionless position by two ropes that each makes a 22.5° angle with the horizontal. What is the tension in the ropes?

First draw the problem as described with the ropes at the correct angles, sign shown and choose a coordinate system.

Then draw the free body diagram of the system, with a dot representing the origin



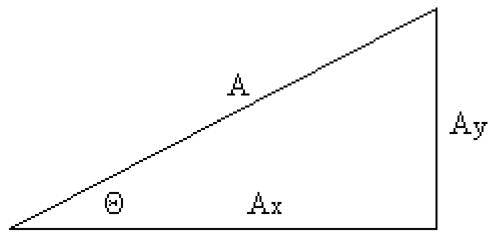
Forces in 2D - Fig. 3

Known	*Unknown*
$\theta = 22.5°$	$F_A = ?$
$Fg = 168$ N	$F_B = ?$

The sum of the two rope forces and the downward weight force is zero. Remember that to do algebraic calculations on vectors, we do not work with the vectors themselves, rather on the components of the vectors – so back to trigonometry we go!

$$Ax = A\cos\Theta;\ \text{therefore}\ \cos\Theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{Ax}{A}$$

$$Ay = A\sin\Theta;\ \text{therefore}\ \sin\Theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{Ay}{A}$$



In the horizontal direction, we know that since the sign is stable that $F_{net,x} = 0$, thus $-F_{A,X} + F_{B,X} = 0$

Which expressed in trig gives us $-F_A\cos\Theta + F_B\cos\Theta = 0$ *or* $F_A\cos\Theta = F_B\cos\Theta$, *so* $F_A = F_B$

As in the horizontal direction, we know the sign is stable in the vertical direction: $F_{net,y} = 0$, thus $F_{A,Y} + F_{B,Y} - Fg = 0$, and again substituting we get

$F_A\sin\Theta + F_B\sin\Theta - F_g = 0$, *knowing that* $F_A = F_B$

$F_A\sin\Theta + F_A\sin\Theta - F_g = 0$ *or* $2F_A\sin\Theta - F_g = 0$ *or*

$2F_A\sin\Theta = F_g$

solving for F_A *we get*

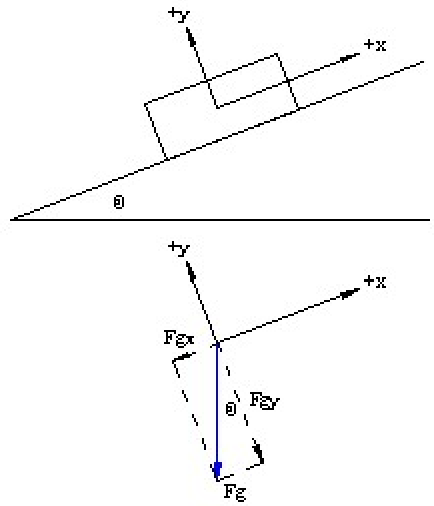
$$F_A = \frac{F_g}{2\sin\Theta} = \frac{168\ N}{2\sin 22.5} = \frac{168\ N}{2\ x\ 0.383} = 2.20\ x\ 10^2\ N$$

The tension in the rope F_A is equal to the tension in rope F_B.

Does the answer make sense?
The answer is positive, as is the direction we choose for the coordinates.
The answer is greater than the weight of the sign – this would be true since the vector in the vertical is only a portion of the resultant vector

- Motion along an Incline Plane
  - The forces due a gravitational field is always directed downward toward the center of the object creating the gravity field – i.e. the planet, moon, etc.
  - When an object is on an inclined plane, the full force of gravity is not applied to the object – rather a lesser amount of gravity is felt according to the angle of the inclined plane. The greater the incline, the less the gravitational force on the object
  - In most inclined plane problems, we tend to rotate the x-axis so that it is parallel to the surface of the inclined plane, and the y-axis is still perpendicular to the x-axis

Sample Problem



Forces in 2D - Fig. 4

A trunk weighing 562 N is resting on a plane inclined 30.0º above the horizontal. Find the components of the weight force parallel and perpendicular to the plane.

Sketch the problem, including a coordinate system (+x-axis pointing up hill as a convention).
Draw the free body diagram showing Fg and the components Fgx and Fgy and the angle θ

Known	*Unknown*
Fg = 562 N	Fgx = ?
Θ = 30.0º	Fgy = ?

Looking at the free body diagram, we see that Fgx and Fgy are negative since they point in directions opposite to the positive axis.

Vector components you will remember are scalar quantities, but they do have signs indicating direction relative to the axis.

So using trig once again,

$$F_{gx} = -F_g \sin\theta$$

$$F_{gx} = -(562\ N)\sin 30.0^\circ = -281\ N$$

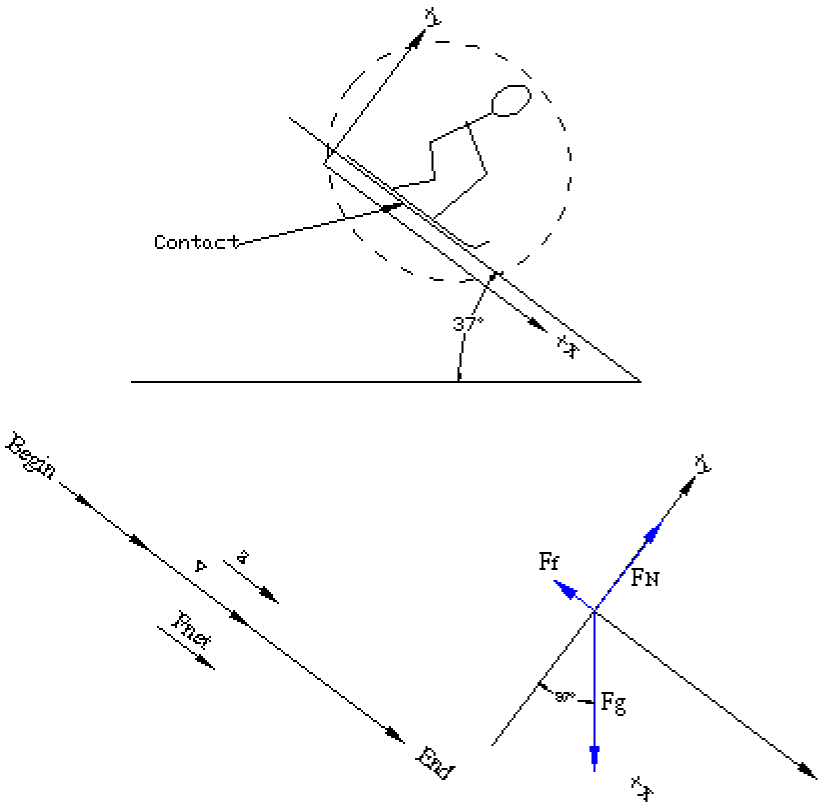
$$F_{gy} = -F_g \cos\theta$$

$$F_{gy} = -(562\ N)\cos 30.0^\circ = -487\ N$$

- That was not too bad, but let's add a twist to it. In the last lesson we learned how to calculate forces with fiction involved – but these were on level surfaces. Now we learn to deal with friction on inclined planes!

Suppose a skier with a mass of 62 kg is going down Death Hill with a slope of 37º. The coefficient of kinetic fiction is 0.15 between the skis and the snow. If the skier starts from rest, how fast will they be going at the end of 5.0 seconds?

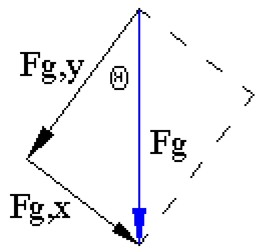
Draw the situation, circle the system and identify the points of contact
Establish a coordinate system
Draw a motion diagram
Draw a free body diagram



Forces in 2D - Fig. 5

Known		*Unknown*
m = 62 kg	$u_k = 0.15$	a = ?
Θ = 37º	vo = 0.0 m/s	v = ?
t = 5.0 s		

In the y-axis there is no acceleration, so the net force is 0. First we solve for the F_N in the y-axis, by setting up the ever-familiar rectangle of forces using Fg as the hypotenuse, and then using sin/cos to solve for the vector components.



Forces in 2D - Fig. 6

$$F_{net,y} = ma_y = 0 = F_N - F_{g,y}$$
$$\textit{which rearranges to}$$
$$F_N = F_{g,y} = F_g \cos\theta = mg \cos\theta$$

In the x-axis, things are a little more complicated. We have an F_{net} greater than zero since the skier is accelerating down the slope, and rather than just using the F_N, we now have to use a portion of it according to the angle of the slope.

According to Newton's 2[nd] law, we know that $a = \frac{F_{net}}{m}$ *or* $F_{net} = ma$. Since we are dealing in the x-axis, this becomes $F_{net,x} = ma_x$, and since all the acceleration is in the x-axis, then we can state that $F_{net,x} = ma_x = ma$. Knowing that $F_{net,x}$ is the sum of the forces in the x-axis, we can substitute for $F_{net,x}$ using F_N – Fg, so we get:

$$F_{gx} - F_f = ma$$
$$\textit{substituting for } F_{gx}$$
$$F_g \sin\theta - F_f = ma$$
$$\textit{substituting } mg \textit{ for } F_g$$
$$mg \sin\theta - F_f = ma$$
$$\textit{substituting } u_k F_N \textit{ for } F_f$$
$$mg \sin\theta - u_k F_N = ma$$
$$\textit{and finally substituting for } F_N \textit{ from before}$$
$$mg \sin\theta - u_k mg \cos\theta = ma$$
$$\textit{dividing both sides by } m$$
$$a = g \sin\theta - u_k g \cos\theta$$
$$\textit{simplifying}$$
$$a = g\,(\sin\theta - u_k \cos\theta)$$

and solving

$$a = 9.80\ m/s^2(\sin 37^\circ - 0.15\cos 37^\circ)$$

$$a = 9.80\ m/s^2(.602 - 0.15 \bullet .799)$$

$$a = 9.80\ m/s^2(.482) = 4.7 m/s^2$$

Now we know that the skier was accelerating at 4.7 m/s^2. We were asked "how fast he was going at the end of 5.0 seconds", so using our old friend for constant acceleration:

$$v = v_o + at$$

$$v = 0\ m/s = 4.7 m/s^2 \bullet 5.0\ s$$

$$v = 23.5\ m/s\ or\ 24\ m/s$$

Practice problems

Consider the trunk on the incline in the first example problem – if the incline were a frictionless surface, what would be the magnitude of its acceleration?

We calculated that the trunk had a force of 281 N working on it due to the force of gravity in the x-axis. Knowing that the trunk had a weight of 562 N – which is 57.3 kg, then we could use

$$a = \frac{F_{net}}{m} = \frac{281\ N}{57.3\ kg} = 4.90\ m/s^2$$

At the end of 4.00 s how fast would it be going?

$$v = v_o + at = 0\ m/s + 4.90\ m/s^2 \bullet 4.00\ s = 19.6\ m/s$$

From the example problem of the skier going down hill, find the x and y components of the weight going down hill.

$$F_{gx} = F_g \sin\theta$$

$$F_{gx} = (608\ N)\sin 37^\circ = 365.66\ N = 3.66\ x\ 10^2\ N$$

$$F_{gy} = F_g \cos\theta$$

$$F_{gy} = (608\ N)\cos 37^\circ = 485.57\ N = 4.86\ x\ 10^2\ N$$

If the same skier were going down a 30º slope, what would be the magnitude of acceleration?

$$a = 9.80\ m/s^2(\sin 30^\circ - 0.15\cos 30^\circ)$$

$$a = 9.80\ m/s^2(.5 - 0.15 \bullet .866)$$

$$a = 9.80\ m/s^2(.367) = 3.59 m/s^2$$

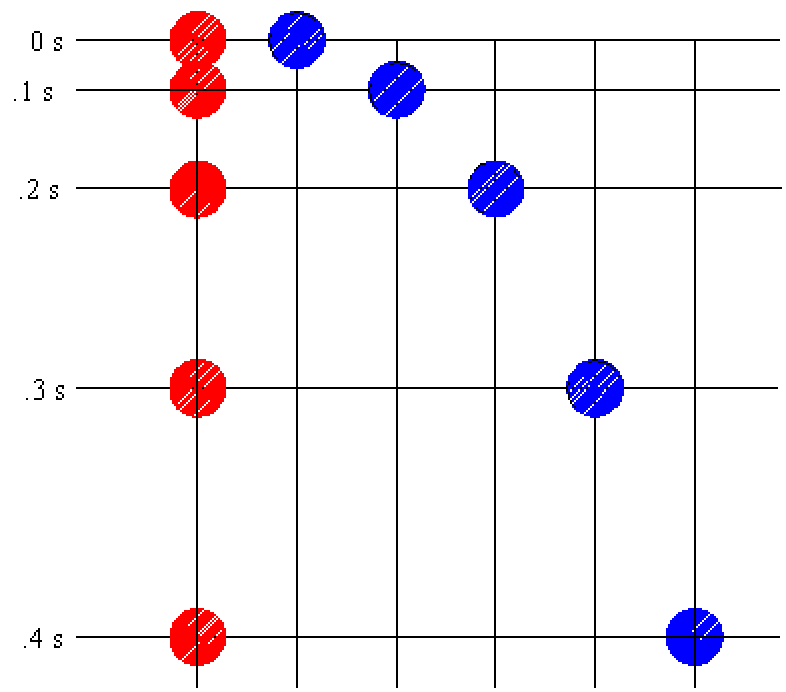
While on the 37º slope, the skier had been accelerating from rest for 5.0 s when he hit a rough spot and the friction between the ski and the snow increased to the point that the net force on the skier became 0. What is the new coefficient of friction, and how fast would the skier be going after going an additional 5 .0 s?

From earlier, we found that
$a = g\ (\sin\theta - u_k\ \cos\theta)$
so if we now know that a = 0, then
$0 = g\ (\sin\theta - u_k\ \cos\theta)$
$0 = g\ \sin\theta - gu_k\ \cos\theta$
$g\ \sin\theta = gu_k\ \cos\theta$
$$u_k = \frac{g\ \sin\theta}{g\cos\theta}$$
$$u_k = \frac{\sin\theta}{\cos\theta}$$
$$u_k = \frac{\sin 37^\circ}{\cos 37^\circ}$$
$$u_k = \frac{.602}{.799}$$
$u_k = .75$

Since a = 0, then the velocity would be the same as just before hitting the rough snow.

Projectile Motion
Objectives: Recognize that the vertical and horizontal motions of a projectile are independent; Relate the height, time in the air, and initial vertical velocity of a projectile using its vertical motion, then determine the range; Explain how the shape of the trajectory of a moving object depends upon the frame of reference from which it is observed.

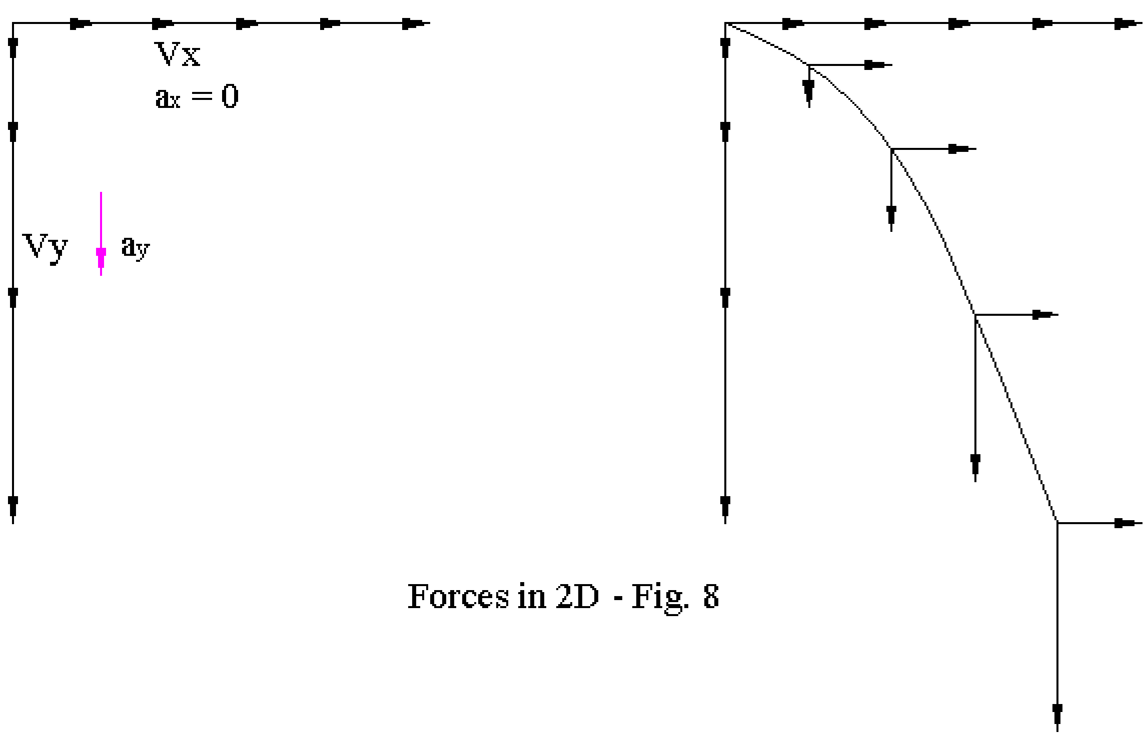
- Projectile – what does that bring to mind?
  - Missile, bullet, but it could be a golf ball, football, baseball, or a balled up piece of paper being thrown toward a trash can
  - A projectile that has been given an initial thrust, ignoring air friction, will move through the air only under the force of gravity.
  - A projectile's course is called it's trajectory
- More Motion Problems, now in 2D
  - Suppose you were to drop two golf balls from the same point at the same time, but one of them you also gave a horizontal force – what would be their trajectories?
    - Which would hit first?
  - As seen from the diagram, they fall at the same rate (Fg), irregardless of the horizontal motion



Forces in 2D - Fig. 7

○

○ Take a look at the motion diagram of the two balls falling, along with a plot of the motion



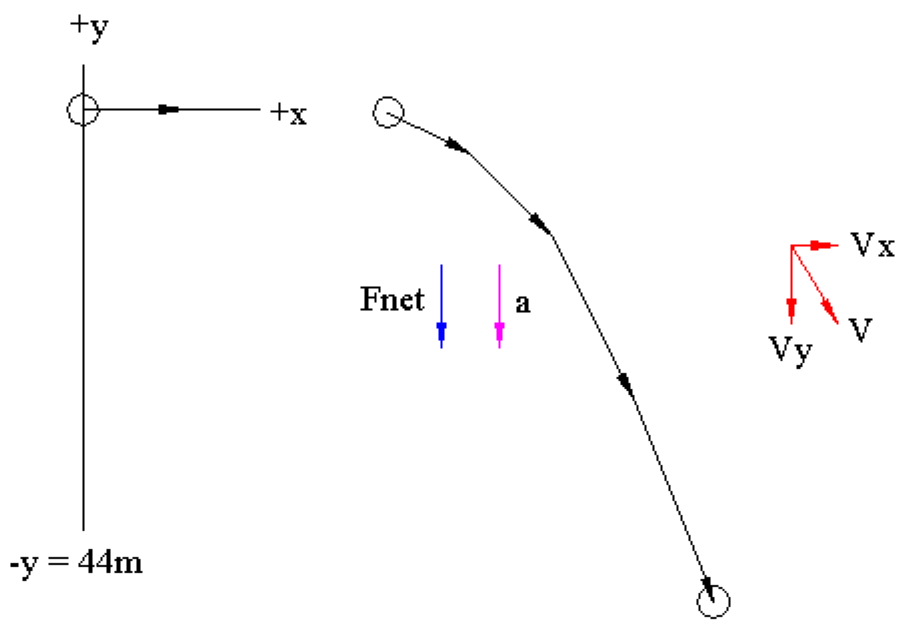
Forces in 2D - Fig. 8

○

- Notice that the vertical motion is the same for both balls(accelerating due to Fg), while the horizontal velocity remains the same.
- By plotting the intersections of the motion diagram we can follow the trajectory of the ball – this has a definite shape, the same shape every time, known as a parabola

Example Problems
You throw a rock off a cliff with a horizontal velocity of 15 m/s. If the cliff is 44 m high, then how far from the base of the cliff will the rock land, and with what velocity will the rock hit the ground?

Begin by drawing the problem, choose a coordinate system, and then add the free body diagram to it



Forces in 2D - Fig. 9

To solve the problem, we need to solve for motion in the x-axis, motion in the y-axis and then combine the two components together.

Known	*Unknown*
$x_0 = 0$ m	x when y = -44 m
$v_{x0} = 15$ m/s	v at that time
y0 = 0 m	
$v_{y0} = 0$ m/s	
a = -g	

Solving for motion in the y-axis, we are bound to use one of our old familiar formulas for uniform motion – to refresh they are:

$$v = v_0 + at$$
$$d = d_0 + 1/2(v + v_0)t$$
$$d = d_0 + v_0 t + 1/2at^2$$
$$v^2 = v_0{}^2 + 2a(d - d_0)$$

We know the distance the rock fell (44 m), the initial velocity (0 m/s), and the acceleration due to gravity (-9.8 m/s^2), so it appears that we can only use $d = d_0 + v_0 t + 1/2at^2$.

$$d = d_0 + v_0 t + \frac{1}{2}at^2$$
dropping out the terms that are 0
$$d = \frac{1}{2}at^2$$
Solving for t gives us
$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(44\ m)}{9.80\ m/s^2}} = \sqrt{8.98\ s^2} = 3.00\ s$$

Lastly lets put the components of the velocity together to get the resultant velocity
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{(29.4\ m/s)^2 + (15\ m/s)^2}$$
$$c = \sqrt{864.36\ m^2/s^2 + 225\ m^2/s^2}$$
$$c = \sqrt{1089.36\ m^2/s^2}$$
$$c = 33.0\ m/s$$

So it took it 3.00 seconds to fall, now let's determine the velocity in the y-axis when it hit
$$v = v_0 + at$$
substituting
$$v = 0\ m/s = -9.8\ m/s^2 \bullet 3.00\ s$$
$$v = 29.4\ m/s$$
Which is the v_y component of the final velocity.

Now let's work on the x-axis, first lets determine how far the rock went from the cliff:

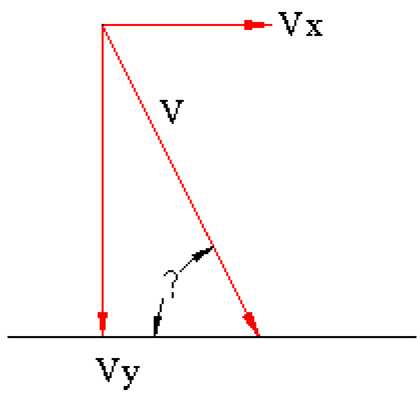
$$d = d_0 + \frac{1}{2}(v + v_0)t$$

substituting

$$d = 0\ m + \frac{1}{2}(15\ m/s + 15\ m/s)3.00\ s$$

$$d = 45.0\ m$$

We could also ask at what angle the rock struck the ground. Knowing the components of the velocity, it is a rather trivial task

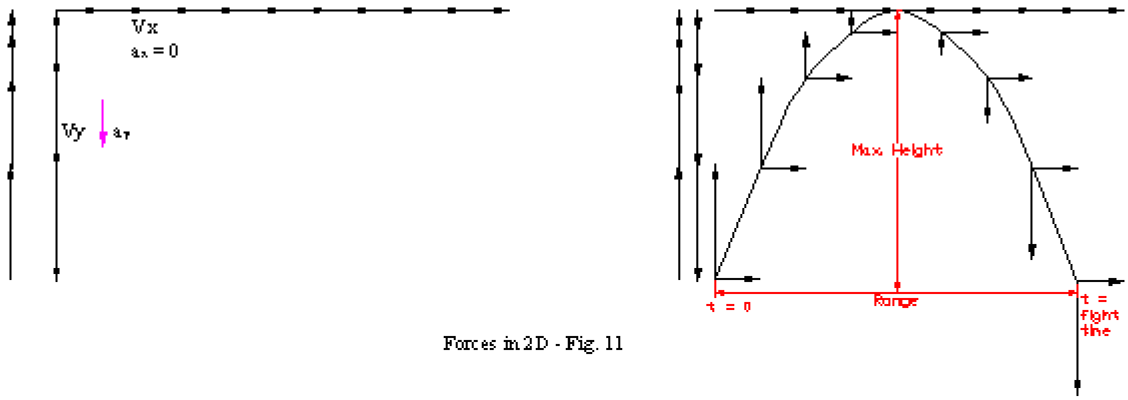


Forces in 2D - Fig. 10

$$\tan\theta = \frac{opp}{adj} = \frac{29.5\ m/s}{15\ m/s} = 1.97$$

taking the arctan *of* 1.97 *yields* 63°

- Projectiles at an angle
    - When launched at an angle, a projectile will have both an initial velocity in the horizontal and the vertical direction
    - The horizontal velocity (neglecting air resistance) will remain constant
    - The vertical velocity, if the motion is upward, will slow to 0, and then begin accelerating downward
    - The vertical velocity, if the motion is downward, will continue to accelerate at the rate of acceleration due to gravity
    - If we look at the trajectory of an object in flight we would get a plot like



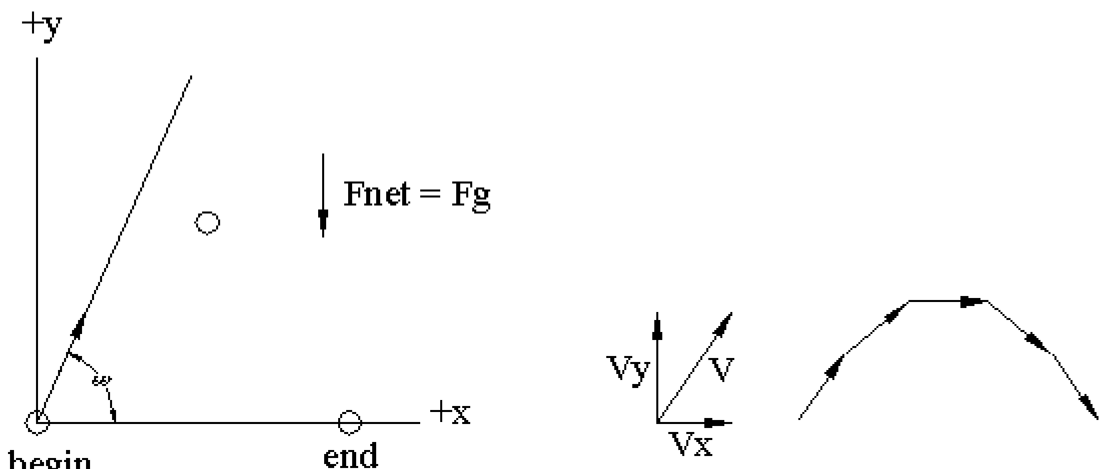
Forces in 2D - Fig. 11

- Several items are named
  - Range – horizontal distance the projectile covers
  - Maximum height – self explanatory
  - Flight time – time the projectile is in the air

- As noted earlier, all free flight projectiles (ignoring air resistance) paths will describe a parabola
- Solving projectile motion problems
  - Choose a coordinate system
  - Draw a basic diagram with coordinate system
  - Draw a motion diagram
  - Draw a free body diagram
  - The motion in the x-axis is solved for independently of motion in the y-axis

Sample Problem

A ball was launched with an initial velocity of 4.47 m/s at an angle of 66º above the horizontal. What was the maximum height, range and time of flight of the ball?



Forces in 2D - Fig. 12

Known
$x_0 = 0$
$y_0 = 0$
$v_0 = 4.47$m/s
$\Theta = 66^o$
a = -9.80 m/s^2

Unknown
y when $v_y = 0$
t = ?
x when y = 0

The range is dependent on velocity in the x-axis and time in the air. Time in the air is dependent on the velocity in the y-axis, so we need to first determine the velocity in the y-axis and x-axis, then solve for motion in the y-axis and then in the x-axis.

Solving for velocities in the x-axis and y-axis

$$v_x = V\cos\theta = 4.47m/s \bullet .4067 = 1.82m/s$$

$$v_y = V\sin\theta = 4.47m/s \bullet .9135 = 4.08m/s$$

Now we can solve for motion in the y-axis. Knowing that the ball reached a velocity in the y-axis when it reached the top of it's trajectory:

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{0m/s - 4.08m/s}{-9.80m/s^2} = .4163s = .42s$$

So the flight time would be twice the time to reach maximum height, or 2 x .42s = .84s

Maximum height would be found by:

$$d = d_0 + 1/2(v_0 + v)t$$

$$d = 0m + \frac{1}{2}(4.08m/s + 0m/s).42s = .8568m = .86m$$

Range would be found by:

$d = d_0 + 1/2\,(v_0 + v)t$

$d = 0m + \frac{1}{2}(1.82m/s + 1.82m/s).84s = 1.53m = 1.5m$

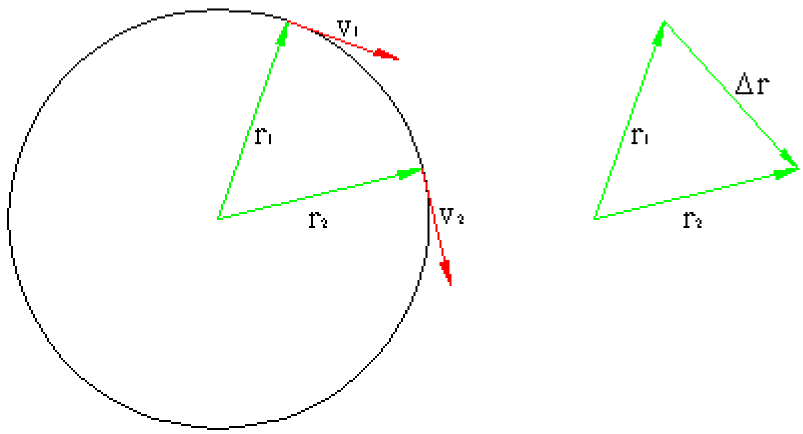
Practice problems

- Trajectories and the Frame of Reference
  - If you were on a moving bus throwing a ball straight up and down, what motion would the ball be making?
    - To you the ball would have only vertical motion, no horizontal motion
    - To an outside observer the ball would have both vertical and horizontal motion. Flight time and maximum height would be the same, but range would be different
- Air Resistance
  - As with previous problems we have neglected air resistance, but it does affect the trajectory of projectiles. For most it is negligible, but for others it can have a major change.
    - Golf ball has dimples that help to maximize the range
    - Spin on a baseball causes it to curve or drop when thrown by a pitcher
    - Lack of spin causes an unpredictable motion by the interaction of air on the lacing of the baseball (knuckle ball)
    - Frisbees generate lift by spinning
    - Paper airplanes (and real airplanes too) generate lift over their wings

Circular motion

Explain the acceleration of an object moving in a circle at constant speed; Describe how centripetal acceleration depends upon the object's speed and the radius of the circle; Recognize the direction of the force that causes centripetal acceleration; Explain how the rate of circular motion is changed by exerting torque on it.

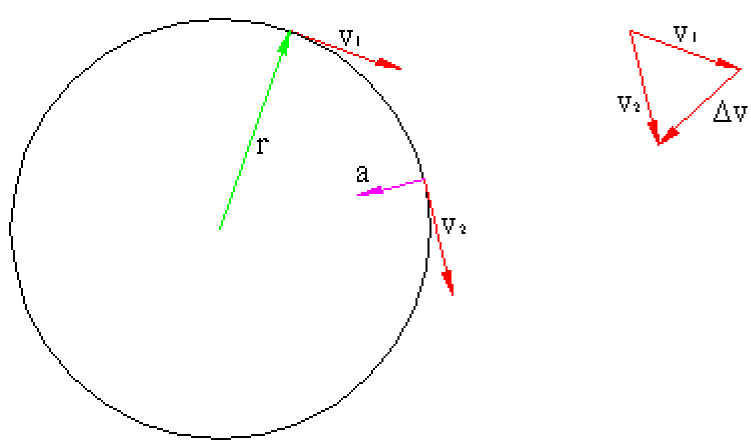
- Circular Motion
  - Can an object have constant speed and changing velocity?
    - Yes – speed if a scalar quantity, whereas velocity is a vector quantity. If the direction was constantly changing (going in a circle) then the speed could be the same, while the velocity (the direction of travel) was changing
  - Look at the motion of an object in circular motion



Forces in 2D - Fig. 13

- Here we have an object at two different positions (r_1 and r_2) as it goes around in a circle. The resultant can be found by subtracting r_1 and r_2 to the change in r – Δr, which takes place during time interval.
- Remember that average velocity was given as $\overline{v} = \frac{\Delta d}{\Delta t}$, for circular motion we can say that average velocity is $\overline{v} = \frac{\Delta r}{\Delta t}$.
- The velocity vector has the same direction as the displacement vector, but a different length. – it is at right angles to the position vector and tangent to its circular path. As the velocity vector moves around the circle, it maintains the same length, but different direction

o Since the velocity is constantly changing, the object has acceleration. Which direction is the object's acceleration?
  - Taking the previous diagram and looking at the velocity vectors we see



Forces in 2D - Fig. 14

- By subtracting the velocity vectors, we get Δv. As before, acceleration is $a = \frac{\Delta v}{\Delta t}$. The acceleration is in the same direction as Δv, which is toward the center of the circle.

- The acceleration of an object in circular motion is always toward the center of the circle and for that reason is called center seeking or **centripetal acceleration**.
- Measuring Centripetal acceleration
  - If we compare the displacement triangle to the velocity triangle, we see that the angles for both are the same – therefore the two triangles are similar triangles, meaning that the ratio of the lengths or two corresponding sides are equal. Thus we can state that $\frac{\Delta r}{r} = \frac{\Delta v}{v}$. Dividing both sides by Δt, we get $\frac{\Delta r}{r\Delta t} = \frac{\Delta v}{v\Delta t}$
  - Since we already have established that $v = \frac{\Delta r}{\Delta t}$ and that $a = \frac{\Delta v}{\Delta t}$, then we can substitute these and get $\frac{v}{r} = \frac{a}{v}$, and solving for a, we get $a_c = \frac{v^2}{r}$. a_c is the notation for centripetal acceleration.
  - As noted, centripetal acceleration always points to the center of circular motion
  - So how does one measure the speed of an object moving in a circle? The simplest is the period T – that is the time for the object to make a complete 360º. The distance that an object will travel in 360º is dependent on the circumference of the circle, which is 2πr. So velocity can be expressed a $v = \frac{2\pi r}{T}$
  - If we do a little more substituting, of this v into the previous equation then we end up with $a_c = \frac{(2\pi r / T)^2}{r}$ or $a_c = \frac{4\pi^2 r}{T^2}$

- How about Forces in circular motion

  - Remember that F_{net} = ma, life in circles is not much different. Now we have the following equations:

$$F_{net} = ma_c$$

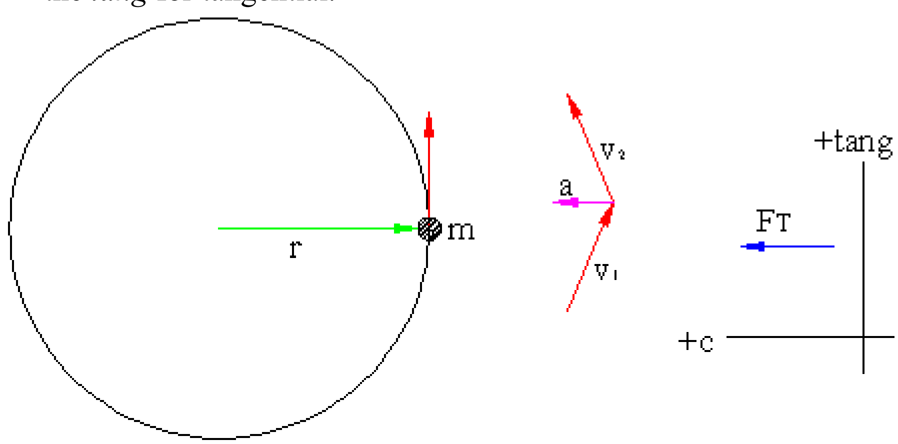
$$\text{substituting where } a_c = \frac{v^2}{r}$$

$$F_{net} = \frac{mv^2}{r}$$

$$\text{or substituting where } a_c = \frac{4\pi^2 r}{T^2}$$

$$F_{net} = m\frac{4\pi^2 r}{T^2}$$

- When choosing a coordinate system for circular motion, it is common to choose one axis as pointing in the direction of acceleration – always toward the center of the circle
- Rather than labeling the axis as x and y, it is more common to call it *c* for centripetal. The other axis, again at right angles is called the *tang* for tangential.



Forces in 2D - Fig. 15

Example Problem

If we have a 0.13g rubber stopper on the end of a 0.93 m string, and swing the stopper around in a horizontal circle with a period of 1.18s, what is the tension in the string exerted by the stopper?

Known	*Unknown*
m = 0.13 g	F_T = ?
T = 1.18 s	
r = 0.93 m	

We know that Fnet = F_T = ma_c, and since we know the mass of the stopper, all we are left to determine is that acceleration of the stopper. Using our new formula for acceleration in a circular motion, we have the following:

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$a_c = \frac{4\pi^2 0.93m}{(1.18s)^2} = \frac{36.68m}{1.39s^2} = 26.34m/s^2 = 26.3m/s^2$$

Knowing the acceleration, we can turn back to our force formula to determine the tension in the string

$$F_T = ma_c$$

$$F_T = 13g(26m/s^2)$$

$$F_T = 1.3\,x\,10^{-3}kg(26m/s^2)$$

$$F_T = .338\,N = .34N$$

- The mythical Centrifugal Force
  - When you are in a car and the car suddenly veers right, which way do you go (left).
  - You have probably grown up being taught that this force that pushes you outward like that is called Centrifugal Force. You are now old enough to know the truth – there is no such thing as centrifugal force!
  - According to Newton's 2^{nd} law, if you are in a car that suddenly veers to the right, what is the natural motion of the body – to continue straight at the same velocity
  - But the door acts against the body in a direction that is toward the direction of acceleration, that is the center of the circle. So there is no "outward" force to cause you to go away from the direction of acceleration.
- You really Torque me!
  - In our consideration of circular motion we have been dealing with point masses (ball on a string, sock in a dryer, even the body in a car), in which there is no physical connection between the center of the circle and the body that is moving
  - How about objects that are **rigid rotating objects**? That is a mass that is rotating around its own axis.
    - Washing machine tub spinning
    - Merry-go-round
    - Revolving door
    - An ordinary door (partial circle)
  - So what is the best way to push a door open?
    - The door rotates around it's axis (the pin in the hinge)
    - Where to apply force to push the door – farthest away from the axis takes less force, also applying the force at right angles to the surface of the door.
    - The rigid surface between the axis of rotation and where the force is applied is called a **lever arm**.

- The product of the lever arm and the applied force is called **torque.**
- Torque is a measurement of rotational motion – the greater the torque the greater the rotational motion.
- Torque can increase rotational motion, stop rotational motion and reverse rotational motion.
- The length of a lever arm is directly proportional to the torque – thus Archimedes said, "Give me a lever long enough and I will move the Earth"
- Since torque is a product of force and distance, then we could state that torque = d x F
  - Take a see-saw: in order for it to balance, there must be equal torque on each side of the fulcrum. In this case the force is due to the mass of a child and the acceleration due to gravity, or

    $$Torque = F \bullet d$$

    $$where\ F = mg,\ so$$

    $$Torque = mgd$$
  - So $F_{net} = 0$ for a balanced see-saw, expanding we get that $F_{net} = 0 = Torque_A - Torque_B$ and simplifying we get that $Torque_A = Torque_{B,}$ substituting we get $m_A g d_A = m_B g d_B$
  - So how does this help us understand how a 90kg parent can see-saw with a 20 kg child?
  - This concept is also used in the triple-beam balance.

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