Physics

Lesson Plan #2
A Mathematical Toolkit
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The Measures of Science

Objectives: Define SI, metric prefixes, estimate measurements, scientific notation

- Math – the language of Physics
- How would you measure without a ruler?
- 1795 the French adopted the Metric System which has become the Susteme Internaionale d'Unites – SI
- Standards are kept at the International Bureau of Weights and Measures in Sevres, France and in the National Institute of Science and Technology (NIST) in Gaithersburg Maryland
- Base Quantities are length, time, mass
    - Length – meter
        - 1/10,000,000 distance from the north pole to the equator
        - Distance between two lines on a platinum-iridium bar
        - Distance light travels in a vacuum in 1/299,792,458 s
    - Time – second
        - 1/86,400 of a mean solar day
        - frequency of cesium-133 atom
    - Mass – kilogram
        - Platinum-iridium alloy cylinder
- Derived units are combinations of the base units.
    - m/sec for speed, $kg{\cdot}m^2/s^2$
- Which is more accurate – metric or English system?
    - Accuracy is the same, metric is based on 10 making it easier to use
- SI Prefixes

Prefix	Symbol	Multiplier	Scientific Notation	Example
femto	f	1/1,000,000,000,000,000	10^{-15}	femtosecond (fs)
pico	p	1/1,000,000,000,000	10^{-12}	picometer (pm)
nano	n	1/1,000,000,000	10^{-9}	nanometer (nm)
micro	μ	1/1,000,000	10^{-6}	microfarad (μF)
milli	m	1/1,000	10^{-3}	millimeter (mm)
centi	c	1/100	10^{-2}	centimeter (cm)
deci	D	1/10	10^{-1}	Deciliter (dL)
kilo	K	1000	10^{3}	Kilometer (km)
mega	M	1,000,000	10^{6}	Megabyte (Mb)
giga	G	1,000,000,000	10^{9}	Gigawatt (Gw)
tera	T	1,000,000,000,000	10^{12}	Terabyte (Tb)

-Examples

- 10 mm = 1cm
- 10 cm = 1 dm
- 10 dm = 1 m
- 1000 m = 1 km

- Scientific Notation
  - Convenient way to express very large or very small numbers
  - $M \times 10^{n}$ where $1 \leq M < 10$ and is multiplied by a whole number power of 10
  - Moving the decimal left, add to n. $1000. = 1000 \times 10^{0} \rightarrow 100 \times 10^{1} \rightarrow 10 \times 10^{2} \rightarrow 1 \times 10^{3}$
  - Moving the decimal right, subtract from n. $.0001 \rightarrow .0001 \times 10^{0} \rightarrow .001 \times 10^{-1} \rightarrow .01 \times 10^{-2} \rightarrow .1 \times 10^{-3} \rightarrow 1 \times 10^{-4}$
  - Avg. distance from the sun to Mars is 227,800,000,000 m $\rightarrow 2.278 \times 10^{11}$ m
  - The mass of an electron is 0.000,000,000,000,000,000,000,000,000,000,911 kg $\rightarrow 9.11 \times 10^{-31}$ kg
  - Some calculators show scientific notation as ME^{n} , students should always write answer in full scientific notation ($M \times 10^{n}$)

- Addition and Subtraction Using Scientific Notation
  - If numbers have the same exponent *n*, then add or subtract the values of M leaving *n* the same.
  - 4×10^{8} m + 3×10^{8} m = 7×10^{8} m
  - 4.1×10^{-6} kg – 3.0×10^{-6} kg = 1.1×10^{-6} kg
  - 4.01×10^{6} m +1.89×10^{2} m = 4.01×10^{6} m - $.000189 \times 10^{6}$ m = 4.01×10^{6} m – do example of 240,00,000 +27 on calculator and on board

- Multiplication and Division Using Scientific Notation
  - Multiply the value and units of M, add exponents *n*.
  - Divide the values and units of M, subtract the exponent *n* of the divisor from the exponent *n* of the dividend
  - (4×10^{3} kg) (5×10^{11} m)
    $4 \times 5 = 20$, kg x m = kg•m, $3 + 11 = 14$
    20×10^{14} kg•m = 2×10^{15} kg•m
  - (8×10^{6} m^{3})/(2×10^{-3} m^{2})
    $8/2 = 4$, $6-(-3) = 9$, $3-2 = 1$
    4×10^{9} m

- Converting Units – Factor-Label Method
  - Convert 465g $\rightarrow$ kg
  - Setup a conversion factor of 1. Knowing that 1 kg = 1000g then we can construct 1 = 1 kg/1000 g or 1 = 1000g/1 kg
  - Multiplying or dividing by 1 does not change a value
  - Set up the conversion such that units cancel out:

- $465g = 465g\left(\frac{1kg}{1000g}\right) = \frac{465g \times 1kg}{1000g} = \frac{465kg}{1000} = 0.465kg$
- If units do not work out, check your conversion factor

- Example Problems
  - 1.1 cm to meters
  - $1.1cm = 1.1cm\left(\frac{1m}{100cm}\right) = \frac{1.1cm \times 1m}{100cm} = \frac{1.1m}{100} = \frac{1.1m}{10^2} = 1.1x10^{-2}m$
  - or $1.1cm = 1.1cm\left(\frac{1m}{10^2 cm}\right) = \frac{1.1cm \times 1m}{10^2 cm} = \frac{1.1m}{10^2} = 1.1x10^{-2}m$
  - or $1.1cm = 1.1cm\left(\frac{10^2 m}{1cm}\right) = \frac{1.1cm \times 10^2 m}{1cm} = \frac{1.1x10^2 m}{1} = 1.1x10^{-2}m$

  - 76.2 pm to mm
  - $76.2pm = 76.2pm\left(\frac{1m}{10^{12} pm}\right)\left(\frac{10^3 mm}{1m}\right) = \frac{76.2pm \times 1m \times 10^3 mm}{10^{12} pm \times 1m} =$
    $\frac{76.2 \times 10^3 mm}{10^{12}} = 76.2 \times 10^{-9} mm = 7.62x10^{-8} mm$

  - $76.2pm = 76.2pm\left(\frac{1mm}{10^9 pm}\right) = \frac{76.2 \times 1mm}{10^9} = \frac{76.2mm}{10^9} =$
    $76.2x10^{-9} mm = 7.62x10^{-8} mm$

- Precision and Accuracy
  - Precision – the exactness of measurement (tolerance)
  - Precision can be ½ the smallest division on the scale
    - Using round off to determine ½
    - On a typical meter stick, 1mm is the smallest division, therefore .5 mm is best precision
    - On a micrometer the smallest division is .01 mm, therefore .005 mm is the best precision
  - Accuracy – how well the results agree with a standard value
    - Instrument must be calibrated to known standard
- Error
  - Error = accepted value – experimental value
    - Can be positive or negative
- Percent Error
  - Percent error =(|error|/accepted value) x 100%
- Significant Digits – non-zero numbers
  - Draw diagram of mm ruler, 8.64 cm bar and a cm ruler

  - Note that the bar is 8.6 mm + a little by the mm ruler
  - The little is estimated to be .4 mm
  - So the length of the bar is 8.64 mm by the mm ruler – 8.6 can be seen, the .04 is an estimate
  - Note that the bar is 8 + a little by the cm ruler
  - The little is estimated to be .6 cm
  - So the length of the bar is 8.6 cm by the cm ruler – 8. can be seen, the .6 is an estimate
  - 8.64 is 3 significant figures, 8.6 is 2 significant figures
  - Redraw to have the bar 8.60 in length – 8.60 is 3 significant figures
- Significant Digits – zero's
  - Not all 0's are significant
  - Place holders are not significant
    - 0.0086 = 2 significant figures
    - 0.00860 = 3 significant figures
    - 186,000 = 3, maybe six – unknown since the decimal is not shown
    - 186 km is 3sf, 186.000 is 6sf
    - 1.86×10^5 has 3 sf
- Rules for Significant Digits
  - Non-zero digits are always significant
  - All final 0's after the decimal point are significant
    - These are 0's after the final non-zero digit
  - Zero between two other significant digits are always significant
  - Zeros used solely as place holders are not significant
- The following examples have 3 significant digits
  - 245 m   18.0 g 308 km   0.00623 g

- Arithmetic with Significant Digits
  - An answer can never be more precise that the least precise number
- Addition & Subtraction
  - Add 24.615 + 3.21 + 6.964 = 34.789
    - Since 3.21 has the least number of digits to the right of the decimal, the correct answer is 34.79
    - Same principle for subtraction
- Multiplication & Division
  - 3.22 cm x 2.1 cm = 6.762 cm^2 ->6.8 cm^2
  - Answer is rounded off to have the same number of significant digits as the factor with the least number of significant digits
  - 36.5 m / 3.414s = 10.691 m/s -> 10.7 m/s